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2011.9.18

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$\sqrt{2}$ の近似分数の作り方について、整理しました。基本的なものと考えられる4つの方法を示しました。

$\sqrt{3}$ の場合 ③を使って

$$\frac{3}{4} \begin{matrix} (\times 4) \\ \hline \end{matrix} \quad \frac{12}{16} \rightarrow 14 \begin{matrix} (-1) \\ (+1) \end{matrix} \quad \frac{13}{15}$$

$$\frac{13}{15} \begin{matrix} (\times 2) \\ \hline \end{matrix} \quad \frac{26}{15} \doteq \sqrt{3}$$

$$\frac{3 \times 15^2}{26^2} = \frac{675}{676} \begin{matrix} (\times 4) \\ \hline \end{matrix} \quad \frac{2700}{2704}$$

$$2702 \begin{matrix} (-1) \\ (+1) \end{matrix} \quad \frac{2701}{2703}$$

2

$$\frac{2701}{2703} \quad (\times 26) \quad \frac{70226}{40545} \stackrel{?}{=} \sqrt{3}$$

$$70226^2 + A = 3 \times 40545^2$$
$$A = -1$$

計算の始まりの決め方は簡単です。

$$\frac{3}{4} \leftarrow 3 \div 4 = 3 \div 2^2$$

$\sqrt{2}$ の近似分数の作りかた

① [一次収束]

$$\frac{1}{1} \quad \frac{2}{1} \quad \frac{3}{2}$$

$$\frac{3}{2} \quad \frac{4}{3} \quad \frac{7}{5} \quad \left(\frac{3+4}{2+3} = \frac{7}{5} \right)$$

$$\frac{7}{5} \quad \frac{10}{7} \quad \frac{17}{12}$$

$$\frac{17}{12} \quad \frac{24}{17} \quad \frac{41}{29}$$

$$\frac{41}{29} \quad \frac{58}{41} \quad \frac{99}{70}$$

$$\frac{99}{70} \quad \frac{140}{99} \quad \frac{239}{169}$$

$$\frac{239}{169} \quad \frac{338}{239} \quad \frac{577}{408}$$

$$\frac{577}{408} \quad \frac{816}{577} \quad \frac{1393}{985}$$

$$\frac{1393}{985} \quad \frac{1970}{1393} \quad \frac{3363}{2378}$$

② [2次収束]

$$2 = 2 \times 1$$

$$\frac{2+1}{2} = \frac{3}{2} \quad 2 \times \frac{2}{3} = \frac{4}{3}$$

$$\frac{\frac{3}{2} + \frac{4}{3}}{2} = \frac{17}{12} \quad 2 \times \frac{12}{17} = \frac{24}{17}$$

$$\frac{\frac{17}{12} + \frac{24}{17}}{2} = \frac{577}{408} \quad 2 \times \frac{408}{577} = \frac{816}{577}$$

$$\frac{\frac{577}{408} + \frac{816}{577}}{2} = \frac{665857}{470832}$$

$$2 \times \frac{470832}{665857} = \frac{941664}{665857}$$

$$\frac{\frac{665857}{470832} + \frac{941664}{665857}}{2}$$

$$= \frac{886731088897}{627013566048}$$

③ [3次収束]

$$\frac{2}{1} \begin{matrix} (x4) \\ (x4) \end{matrix} \quad \frac{8}{4} \quad \frac{8+4}{2} = 6 \begin{matrix} (+1) \\ (-1) \end{matrix} \quad \frac{7}{5}$$

$$2 \times \frac{5^2}{7^2} = \frac{50}{49} \begin{matrix} (x4) \\ (x4) \end{matrix} \quad \frac{200}{196} \rightarrow 198$$

$$198 \begin{matrix} (+1) \\ (-1) \end{matrix} \quad \frac{199}{197} \begin{matrix} (x7) \\ (x5) \end{matrix} \quad \frac{1393}{985}$$

$$2 \times \frac{985^2}{1393^2} = \frac{1940450}{1940449} \begin{matrix} (x4) \\ (x4) \end{matrix}$$

$$\frac{7761800}{7761796} \rightarrow 7761798 \begin{matrix} (+1) \\ (-1) \end{matrix}$$

$$\frac{7761799}{7761797} \begin{matrix} (x1393) \\ (x985) \end{matrix} \quad \frac{10812186007}{7645370045}$$

④ [ボンバリ式連分数]

$$2 = 1^2 + 1$$

$$\sqrt{2} = 1 + \frac{1}{2+}$$

$$\frac{1}{0} \quad \frac{1}{1} \quad \frac{3}{2} \quad \frac{7}{5} \quad \frac{17}{12} \quad \frac{41}{29}$$

$$\frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2}$$

$$\frac{99}{70} \quad \frac{239}{169} \quad \frac{577}{408} \quad \frac{1393}{985} \quad \frac{3363}{2378}$$

$$\left(\begin{array}{l} 7 \times 1 + 17 \times 2 = 41 \\ 5 \times 1 + 12 \times 2 = 29 \end{array} \right)$$

$\sqrt{2}$ の近似分数を求める簡単な計算法

$\frac{a}{b}$	$2 \times \frac{b}{a}$	$\frac{a+2b}{b+a}$
$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{2}$
$\frac{3}{2}$	$\frac{4}{3}$	$\frac{7}{5}$
$\frac{7}{5}$	$\frac{10}{7}$	$\frac{17}{12}$
$\frac{17}{12}$	$\frac{24}{17}$	$\frac{41}{29}$
$\frac{41}{29}$	$\frac{58}{41}$	$\frac{99}{70}$
$\frac{99}{70}$	$\frac{140}{99}$	$\frac{239}{169}$
$\frac{239}{169}$	$\frac{338}{239}$	$\frac{577}{408}$

(分子)² + A = 2 × (分母)²
 の A を使って分類します。

A	1	-2	-1	2
$\frac{1}{1}$	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{4}{3}$	
$\frac{7}{5}$	$\frac{10}{7}$	$\frac{17}{12}$	$\frac{24}{17}$	
$\frac{41}{29}$	$\frac{58}{41}$	$\frac{99}{70}$	$\frac{140}{99}$	
$\frac{239}{169}$	$\frac{338}{239}$	$\frac{577}{408}$		

$\sqrt{3}$ の場合

$\frac{1}{1}$	$\frac{3}{1}$	$\frac{4}{2} = \frac{2}{1}$
$\frac{2}{1}$	$\frac{3}{2}$	$\frac{5}{3}$
$\frac{5}{3}$	$\frac{9}{5}$	$\frac{14}{8} = \frac{7}{4}$
$\frac{7}{4}$	$\frac{12}{7}$	$\frac{19}{11}$
$\frac{19}{11}$	$\frac{33}{19}$	$\frac{52}{30} = \frac{26}{15}$
$\frac{26}{15}$	$\frac{45}{26}$	$\frac{71}{41}$
$\frac{71}{41}$	$\frac{123}{71}$	$\frac{194}{112} = \frac{97}{56}$
$\frac{97}{56}$ を使って		
$\frac{97}{56}$	$\frac{168}{97}$	$\frac{265}{153}$
$97 \times 14 - 7 = 1351$		$\frac{1351}{780}$
$56 \times 14 - 4 = 780$		

$\sqrt{5}$ の場合

$\frac{1}{1}$	$\frac{5}{1}$	$\frac{6}{2} = \frac{3}{1}$
$\frac{3}{1}$	$\frac{5}{3}$	$\frac{8}{4} = \frac{2}{1}$
$\frac{2}{1}$	$\frac{5}{2}$	$\frac{7}{3}$
$\frac{7}{3}$	$\frac{15}{7}$	$\frac{22}{10} = \frac{11}{5}$
$\frac{11}{5}$	$\frac{25}{11}$	$\frac{36}{16} = \frac{9}{4}$
$\frac{9}{4}$	$\frac{20}{9}$	$\frac{29}{13}$
$\frac{29}{13}$	$\frac{65}{29}$	$\frac{94}{42} = \frac{47}{21}$
$\frac{47}{21}$	$\frac{105}{47}$	$\frac{152}{68} = \frac{38}{17}$

加速法 (√2の場合)

2 = 2 × 1

$\frac{2+1}{2} = \frac{3}{2}$ $2 \times \frac{2}{3} = \frac{4}{3}$

$\frac{\frac{3}{2} + \frac{4}{3}}{2} = \frac{9+8}{12} = \frac{17}{12}$

$2 \times \frac{12}{17} = \frac{24}{17}$

$\frac{\frac{17}{12} + \frac{24}{17}}{2} = \frac{289+288}{408} = \frac{577}{408}$

A -1 2

$\frac{3}{2}$
 $\frac{17}{12}$
 $\frac{577}{408}$

√2 > $\frac{2}{3}$ の求め方

$\frac{2}{1}$ $\frac{2}{5}$

$\frac{2}{1}$ $\frac{6+1}{6-1} = \frac{7}{5}$

$\frac{2}{1}$ $\frac{6}{6-1} = \frac{6}{5}$

$\frac{2}{1}$ $\frac{8+4}{2} = 6$ $\frac{6+1}{6-1} = \frac{7}{5}$

$\frac{2}{1} \times 4 = \frac{8}{4}$ $\frac{8+4}{2} = 6$ $\frac{6+1}{6-1} = \frac{7}{5}$

$\frac{\frac{2}{5} + \frac{10}{7}}{2} = \frac{99}{70} > \sqrt{2}$

√3の場合

$\frac{3}{1}$ $\frac{5}{3}$

$\frac{3}{1}$ $\frac{4+1}{4-1} = \frac{5}{3}$

$\frac{3}{1}$ 4 $\frac{4+1}{4-1} = \frac{5}{3}$

$\frac{3}{1}$ $\frac{6+2}{2} = 4$ $\frac{4+1}{4-1} = \frac{5}{3}$

$\frac{3}{1} \times 2 = \frac{6}{2}$ $\frac{6+2}{2} = 4$ $\frac{4+1}{4-1} = \frac{5}{3}$

$\frac{7}{9} \times 2 = \frac{14}{18}$ 16 $\frac{15}{17} \times 3 = \frac{45}{17}$

$(\frac{45}{17})^2 = 7.006920412$

√5の場合

$\frac{5}{1} \times 1 = \frac{5}{1}$ $\frac{5+1}{2} = 3$ $\frac{4}{2} = \frac{2}{1}$

$\frac{5}{4} \times 4 = \frac{20}{4}$ 18 $\frac{19}{17} \times 2 = \frac{38}{17}$

$\frac{10}{9} \times 4 = \frac{40}{9}$ 38 $\frac{39}{37} \times 3 = \frac{117}{37}$

$(\frac{117}{37})^2 = 9.999269538$

$\frac{\frac{2}{1} + \frac{5}{2}}{2} = \frac{9}{4}$ $\frac{2}{1} < \sqrt{5} < \frac{9}{4}$

$\frac{\frac{3}{1} + \frac{10}{3}}{2} = \frac{19}{6}$ $\frac{3}{1} < \sqrt{10} < \frac{19}{6}$