

第 4 章 (案)

階差数列について

規則性をみつけることで式を大量に効率的に求めることができます。

8乗数の数列の階差の表づくり

CASIO FX-890P

Function Memory を使用

- M-8-0 N^8
- M-8-1 $\boxed{1}$ $N^8 - (N-1)^8$
① $(N+1)^8 - N^8$
- M-8-2 $\boxed{2}$ $N^8 - 2(N-1)^8 + (N-2)^8$
② $(N+2)^8 - 2(N+1)^8 + N^8$

| | | | | | |
|-------|--------------------|---------------------|----------------------|-----------------------|-------------------|
| M-8-0 | 0 | 1 | 256 | 6561 | 65536 |
| M-8-1 | $\boxed{1}$ | $\textcircled{255}$ | 6305 | 58975 | |
| M-8-2 | $\boxed{2}$ | 254 | $\textcircled{6050}$ | 52670 | 266114 |
| | $\boxed{-252}$ | 252 | 5796 | $\textcircled{46620}$ | 213444 |
| | $\boxed{1}$ | $\boxed{2}$ | $\boxed{3}$ | $\boxed{4}$ | $\boxed{5}$ |
| | $\textcircled{-2}$ | $\textcircled{-1}$ | $\textcircled{0}$ | $\textcircled{1}$ | $\textcircled{2}$ |

- $\boxed{7}$ M-8-0 N^8
- M-8-1 $N^8 - (N-1)^8$
- M-8-2 $N^8 - (N-1)^8 - \{(N-1)^8 - (N-2)^8\}$
 $= N^8 - (N-1)^8 - (N-1)^8 + (N-2)^8$
 $= N^8 - 2(N-1)^8 + (N-2)^8$
- M-8-3 $N^8 - 2(N-1)^8 + (N-2)^8$
 $- \{(N-1)^8 - 2(N-2)^8 + (N-3)^8\}$
 $= N^8 - 2(N-1)^8 + (N-2)^8$
 $- (N-1)^8 + 2(N-2)^8 + (N-3)^8$
 $= N^8 - 3(N-1)^8 + 3(N-2)^8 + (N-3)^8$
- M-8-4 $N^8 - 4(N-1)^8 + 6(N-2)^8 - 4(N-3)^8 + (N-4)^8$
- M-8-5 $N^8 - 5(N-1)^8 + 10(N-2)^8 - 10(N-3)^8 + 5(N-4)^8 + (N-5)^8$

- M-8-6 $N^8 - 6(N-1)^8 + 15(N-2)^8 - 20(N-3)^8$
 $+ 15(N-4)^8 - 6(N-5)^8 + (N-6)^8$
- ① M-8-0 N^8
- M-8-1 $(N+1)^8 - N^8$
- M-8-2 $(N+2)^8 - 2(N+1)^8 + N^8$
- M-8-3 $(N+3)^8 - 3(N+2)^8 + 3(N+1)^8 - N^8$
- M-8-4 $(N+4)^8 - 4(N+3)^8 + 6(N+2)^8$
 $- 4(N+1)^8 + N^8$
- M-8-5 $(N+5)^8 - 5(N+4)^8 + 10(N+3)^8$
 $- 10(N+2)^8 + 5(N+1)^8 - N^8$
- M-8-6 $(N+6)^8 - 6(N+5)^8 + 15(N+4)^8$
 $- 20(N+3)^8 + 15(N+2)^8 - 6(N+1)^8 + N^8$

8乗数の数列の階差の表

| (N) | M-8-0 | M-8-1 | M-8-2 | M-8-3 | M-8-4 | M-8-5 | M-8-6 | M-8-7 | M-8-8 |
|-----|------------|------------|------------|------------|-----------|----------|---------|---------|-------|
| 1 | 1 | 255 | 2 | -252 | 5544 | -35280 | 90720 | -100800 | 40320 |
| 2 | 256 | 6305 | 254 | 252 | 504 | -5040 | 30240 | -60480 | 40320 |
| 3 | 6561 | 58975 | 6050 | 5796 | 5544 | 5040 | 10080 | -20160 | 40320 |
| 4 | 65536 | 325089 | 52670 | 46620 | 40824 | 35280 | 30240 | 20160 | 40320 |
| 5 | 390625 | 1288991 | 266114 | 213444 | 166824 | 126000 | 90720 | 60480 | 40320 |
| 6 | 1679616 | 4085185 | 963902 | 697788 | 484344 | 317520 | 191520 | 100800 | 40320 |
| 7 | 5764801 | 11012415 | 2796194 | 1832292 | 1134504 | 650160 | 332640 | 141120 | 40320 |
| 8 | 16777216 | 26269505 | 6927230 | 4131036 | 2298744 | 1164240 | 514080 | 181440 | 40320 |
| 9 | 43046721 | 56953279 | 15257090 | 8329860 | 4198824 | 1900080 | 735840 | 221760 | 40320 |
| 10 | 100000000 | 114358881 | 30683774 | 15426684 | 7096824 | 2898000 | 997920 | 262080 | 40320 |
| 11 | 214358881 | 215622815 | 57405602 | 26721828 | 11295144 | 4198320 | 1300320 | 302400 | 40320 |
| 12 | 429981696 | 385749025 | 83858332 | 43858332 | 17136504 | 5841360 | 1643040 | 342720 | 40320 |
| 13 | 815730721 | 660058335 | 101263934 | 68862276 | 25003944 | 7867440 | 2026080 | 383040 | 40320 |
| 14 | 1475789056 | 1087101569 | 170126210 | 88862276 | 35320824 | 10316880 | 2449440 | 423360 | 40320 |
| 15 | 2562890625 | 1732076671 | 274309310 | 104183100 | 48550824 | 13230000 | 2913120 | 463680 | 40320 |
| 16 | 4294967296 | 2680790145 | 385749025 | 152733924 | 65197944 | 16647120 | 3417120 | 504000 | 40320 |
| 17 | 6975757441 | 4044203135 | 427043234 | 217931868 | 85806504 | 20608560 | 3961440 | 544320 | 40320 |
| | | 5963602465 | 644975102 | 303738372 | 110961144 | 25154640 | 4546080 | 584640 | 40320 |
| | | 8616436959 | 948713474 | 383738372 | 141286824 | 30325680 | 5171040 | 624960 | 40320 |
| | | | 1363412990 | 414699516 | 177448824 | 36162000 | 5836320 | 665280 | 40320 |
| | | | 1919399330 | 555986340 | 220152744 | 42703920 | 6541920 | 705600 | 40320 |
| | | | 2652834494 | 733435164 | 270144504 | 49991760 | 7287840 | 745920 | 40320 |
| | | | 3606422402 | 953587908 | | | | | |
| | | | 4830154814 | 1223732412 | | | | | |

多項式の和の形による

数列の分析の例

階差数列の一般式について

K^M の階差数列

M=1の場合

| | | | | | | |
|---|---|---|---|---|---|-------|
| 0 | 1 | 2 | 3 | 4 | 5 | M-1-0 |
| | 1 | 1 | 1 | 1 | 1 | M-1-1 |
| | 0 | 0 | 0 | 0 | | |

M-1-0

N

M-1-1

1

M=2の場合

| | | | | | | |
|---|---|---|---|----|----|-------|
| 0 | 1 | 4 | 9 | 16 | 25 | M-2-0 |
| | 1 | 3 | 5 | 7 | 9 | M-2-1 |
| | | 2 | 2 | 2 | 2 | M-2-2 |
| | | | 0 | 0 | 0 | |

M-2-0

N^2

M-2-1

$2N + 1$

M-2-2

2

M=3の場合

| | | | | | | |
|---|---|---|----|----|-----|-------|
| 0 | 1 | 8 | 27 | 64 | 125 | M-3-0 |
| | 1 | 7 | 19 | 37 | 61 | M-3-1 |
| | | 6 | 12 | 18 | 24 | M-3-2 |
| | | | 6 | 6 | 6 | M-3-3 |
| | | | | 0 | 0 | |

M-3-0

N^3

M-3-1

$3N^2 + 3N + 1$

M-3-2

$6N + 6$

M-3-3

6

M=4の場合

| | | | | | | | |
|----|----|-----|-----|-----|-----|------|-------|
| 0 | 1 | 16 | 81 | 256 | 625 | 1296 | M-4-0 |
| 1 | 15 | 65 | 175 | 369 | 671 | | M-4-1 |
| 14 | 50 | 110 | 194 | 302 | | | M-4-2 |
| | 36 | 60 | 84 | 108 | | | M-4-3 |
| | | 24 | 24 | 24 | | | M-4-4 |
| | | 0 | 0 | | | | |

M-4-0

N^4

M-4-1

$4N^3 + 6N^2 + 4N + 1$

M-4-2

$12N^2 + 24N + 14$

M-4-3

$24N + 36$

M-4-4

24

M-1-0 N

M-1-1 1

M-2-0 N^2

M-2-1 $2N + 1$

M-2-2 2

M-3-0 N^3

M-3-1 $3N^2 + 3N + 1$

M-3-2 $6N + 6$

M-3-3 6

M-4-0 N^4

M-4-1 $4N^3 + 6N^2 + 4N + 1$

M-4-2 $12N^2 + 24N + 14$

M-4-3 $24N + 36$

M-4-4 24

階差数列の一般式の規則性

M=4の場合

M-4-0 N^4

M-4-1 $4N^3 + 6N^2 + 4N + 1$

M-4-2 $12N^2 + 24N + 14$

M-4-3 $24N + 36$

M-4-4 24

※1項 M-4-1は

$N^4 \quad (N+1)^4 - N^4 = 4N^3 + 6N^2 + 4N + 1$

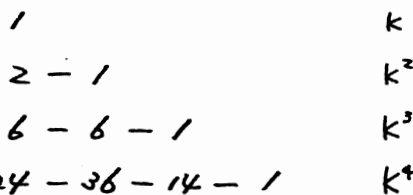
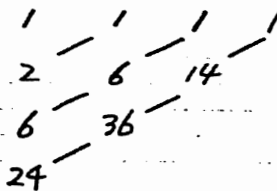
$4N^3 \quad (N+1)^4 - N^4 = 4N^3 + 6N^2 + 4N + 1$

$12N^2 \quad \begin{matrix} 4 & 6 & 4 & 1 \\ \swarrow & \downarrow & \swarrow & \downarrow \\ 4 \times 1 & 6 \times 1 & 4 \times 1 & 1 \times 1 \end{matrix}$

$24N \quad \begin{matrix} 6 \times 2 & 4 \times 6 & 1 \times 14 \end{matrix}$

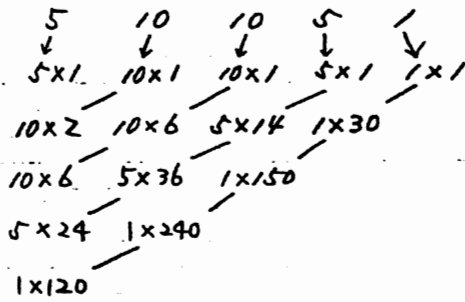
微分を $4 \times 6 \quad 1 \times 36$

くり返す 1×24



の階差0項数列

M=5



- / k
- 2 - / k²
- 6 - 6 - / k³
- 24 - 36 - 14 - / k⁴
- 120 - 240 - 150 - 30 - / k⁵

の階差0項数列

M-5-0

N^5

M-5-1

$$5N^4 + 10N^3 + 10N^2 + 5N + 1$$

M-5-2

$$20N^3 + 60N^2 + 70N + 30$$

M-5-3

$$60N^2 + 180N + 150$$

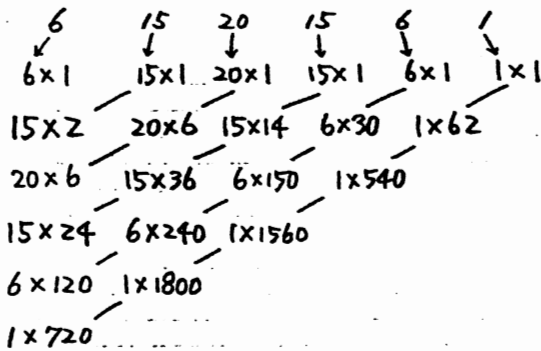
M-5-4

$$120N + 240$$

M-5-5

$$120$$

M=6



- /
- 2 - /
- 6 - 6 - /
- 24 - 36 - 14 - /
- 120 - 240 - 150 - 30 - /
- 720 - 1800 - 1560 - 540 - 62 - /

M-6-0

N^6

M-6-1

$$6N^5 + 15N^4 + 20N^3 + 15N^2 + 6N + 1$$

M-6-2

$$30N^4 + 120N^3 + 210N^2 + 180N + 62$$

M-6-3

$$120N^3 + 540N^2 + 900N + 540$$

M-6-4

$$360N^2 + 1440N + 1560$$

M-6-5

$$720N + 1800$$

M-6-6

$$720$$

M=7 の場合

| | | | | | | | |
|--|---|-----|------|------|-------|-------|------|
| | 1 | | | | | | |
| | 1 | 1 | | | | | |
| | 1 | 2 | | | | | |
| | 1 | 3 | 2 | | | | |
| | 1 | 6 | 6 | | | | |
| | 1 | 7 | 12 | 6 | | | |
| | 1 | 14 | 36 | 24 | | | |
| | 1 | 15 | 50 | 60 | 24 | | |
| | 1 | 30 | 150 | 240 | 120 | | |
| | 1 | 31 | 180 | 370 | 360 | 120 | |
| | 1 | 62 | 540 | 1560 | 1800 | 720 | |
| | 1 | 63 | 602 | 2100 | 3360 | 3520 | 720 |
| | 1 | 126 | 1806 | 8400 | 16800 | 15120 | 5040 |

$$(N+1)^7 = N^7 + 7N^6 + 21N^5 + 35N^4 + 35N^3 + 21N^2 + 7N + 1$$

$$(N+1)^7 - N^7 = 7N^6 + 21N^5 + 35N^4 + 35N^3 + 21N^2 + 7N + 1$$

| | | | | | | | |
|------|-------|-------|-------|-------|-------|------|------|
| | (x7) | (x21) | (x35) | (x35) | (x21) | (x7) | (x1) |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 6 | 14 | 30 | 62 | 126 | | |
| 6 | 36 | 150 | 540 | 1806 | | | |
| 24 | 240 | 1560 | 8400 | | | | |
| 120 | 1800 | 16800 | | | | | |
| 720 | 15120 | | | | | | |
| 5040 | | | | | | | |

N^7 M-7-0

$7N^6 + 21N^5 + 35N^4 + 35N^3 + 21N^2 + 7N + 1$ M-7-1

$42N^5 + 210N^4 + 490N^3 + 630N^2 + 434N + 126$ M-7-2

$210N^4 + 1260N^3 + 3150N^2 + 3780N + 1806$ M-7-3

$840N^3 + 5040N^2 + 10920N + 8400$ M-7-4

$2520N^2 + 12600N + 16800$ M-7-5

$5040N + 15120$ M-7-6

5040 M-7-7

階差数列の一般式の使い方と規則性

$2N^2 + 3N + 4$ の場合

| | | | | | | | | | | |
|-----|---|---|----|----|----|----|----|-----|-----|-----|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| C-0 | 4 | 9 | 18 | 31 | 48 | 69 | 94 | 123 | 156 | 193 |
| C-1 | | 5 | 9 | 13 | 17 | 21 | 25 | 29 | 33 | 37 |
| C-2 | | | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |

| | |
|-----|-----------------|
| C-0 | $2N^2 + 3N + 4$ |
| C-1 | $4N + 5$ |
| C-2 | 4 |

$2 + 3 = 5$ ①

$3N^3 + 4N^2 + 5N + 6$

| | | | | | | | |
|-----|---|----|----|-----|-----|-----|-----|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| C-0 | 6 | 18 | 56 | 138 | 282 | 506 | 828 |
| C-1 | | 12 | 38 | 82 | 144 | 224 | 322 |
| C-2 | | | 26 | 44 | 62 | 80 | 98 |
| C-3 | | | | 18 | 18 | 18 | 18 |

| | |
|-----|------------------------|
| C-0 | $3N^3 + 4N^2 + 5N + 6$ |
| C-1 | $9N^2 + 17N + 12$ |
| C-2 | $18N + 26$ |
| C-3 | 18 |

$3 + 4 + 5 = 12$ ①
 $9 + 17 = 26$ ①
 $3 \times 3 + 4 \times 2 = 17$ ①

$6N^4 + 5N^3 + 4N^2 + 3N + 2$

| | | |
|---------------------------|-------|----|
| $24N^3 + 36N^2 + 24N + 6$ | M-4-1 | x6 |
| $15N^2 + 15N + 5$ | M-3-1 | x5 |
| $8N + 4$ | M-2-1 | x4 |
| 3 | M-1-1 | x3 |

$24N^3 + 51N^2 + 47N + 18$ C-1

| | | | | | |
|-----|---|----|-----|-----|------|
| | 0 | 1 | 2 | 3 | 4 |
| C-0 | 2 | 20 | 160 | 668 | 1934 |
| C-1 | | 18 | 140 | 508 | 1266 |
| C-2 | | | 122 | 368 | 758 |
| C-3 | | | | 246 | 390 |
| C-4 | | | | | 144 |

| | | |
|----------------------|-------|----|
| $72N^2 + 144N + 84$ | M-4-2 | x6 |
| $30N + 30$ | M-3-2 | x5 |
| 8 | M-2-2 | x4 |
| $72N^2 + 174N + 122$ | C-2 | |

| | |
|-----|-------------------------------|
| C-0 | $6N^4 + 5N^3 + 4N^2 + 3N + 2$ |
| C-1 | $24N^3 + 51N^2 + 47N + 18$ |
| C-2 | $72N^2 + 174N + 122$ |
| C-3 | $144N + 246$ |
| C-4 | 144 |

$6 \times 4 + 5 \times 3 + 4 \times 2 = 47$ ①
 $24 \times 3 + 51 \times 2 = 174$ ①
 $51 - 5 \times 3 = 36$
 $36 \div 6 = 6$ } $\rightarrow 6 \times 6 + 5 \times 3 = 51$ ②

M=8 の階差数列 a - 解式 a 表

$$8N^7 + 28N^6 + 56N^5 + 70N^4 + 56N^3 + 28N^2 + 8N + 1$$

$$56N^6 + 336N^5 + 980N^4 + 1680N^3 + 1736N^2 + 1008N + 254$$

$$336N^5 + 2520N^4 + 8400N^3 + 15120N^2 + 14448N + 5796$$

$$1680N^4 + 13440N^3 + 43680N^2 + 67200N + 40824$$

$$6720N^3 + 50400N^2 + 134400N + 126000$$

$$20160N^2 + 120960N + 191520$$

$$40320N + 141120$$

$$40320$$

$$8 \times 35 + 28 \times 20 + 56 \times 10 + 70 \times 4 = 1680 \quad \text{--- (3)}$$

$$8 \times 21 + 28 \times 15 + 56 \times 10 + 70 \times 6 + 56 \times 3 = 1736 \quad \text{--- (2)}$$

$$8 \times 35 + 28 \times 15 + 56 \times 5 = 980 \quad \text{--- (4)}$$

$$\textcircled{5} \quad 8 \times 21 + 28 \times 6 = 336 \quad (N^5)$$

$$\textcircled{4} \quad 8 \times 35 + 28 \times 15 + 56 \times 5 = 980 \quad (N^4)$$

$$\textcircled{3} \quad 8 \times 35 + 28 \times 20 + 56 \times 10 + 70 \times 4 = 1680 \quad (N^3)$$

$$\textcircled{2} \quad 8 \times 21 + 28 \times 15 + 56 \times 10 + 70 \times 6 + 56 \times 3 = 1736 \quad (N^2)$$

$$\textcircled{1} \quad 8 \times 7 + 28 \times 6 + 56 \times 5 + 70 \times 4 + 56 \times 3 + 28 \times 2 = 1008 \quad (N)$$

$$\textcircled{0} \quad 8 + 28 + 56 + 70 + 56 + 28 + 8 = 254$$

| | | | | | |
|------------|---|----|----|-----|-----|
| N^0 (定数) | 1 | 1 | 1 | 1 | 1 |
| N^1 | 2 | 3 | 4 | 5 | 6 |
| N^2 | 3 | 6 | 10 | 15 | 21 |
| N^3 | 4 | 10 | 20 | 35 | 56 |
| N^4 | 5 | 15 | 35 | 70 | 126 |
| N^5 | 6 | 21 | 56 | 126 | 252 |

パスカルの三角形

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|----|----|----|----|----|-----|-----|------|------|------|------|------|------|
| 1 | | | | | | | | | | | | | | | | | | | | |
| | 1 | | | | | | | | | | | | | | | | | | | |
| | | 1 | 1 | | | | | | | | | | | | | | | | | |
| | | | 1 | 2 | 1 | | | | | | | | | | | | | | | |
| | | | | 1 | 3 | 3 | 1 | | | | | | | | | | | | | |
| | | | | | 1 | 4 | 6 | 4 | 1 | | | | | | | | | | | |
| | | | | | | 1 | 5 | 10 | 10 | 5 | 1 | | | | | | | | | |
| | | | | | | | 1 | 6 | 15 | 20 | 15 | 6 | 1 | | | | | | | |
| | | | | | | | | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | | | | | |
| | | | | | | | | | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | | | | |
| | | | | | | | | | | 1 | 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | | |
| | | | | | | | | | | | 1 | 45 | 120 | 210 | 252 | 210 | 120 | 45 | | |
| | | | | | | | | | | | | 1 | 165 | 330 | 462 | 330 | 165 | | | |
| | | | | | | | | | | | | | 1 | 165 | 495 | 792 | 495 | 165 | | |
| | | | | | | | | | | | | | | 1 | 1287 | 1287 | 1287 | 1287 | 1287 | 1287 |