

# 第5章

## 三角数, 台形数について (案)

パスカルの三角形の拡張について  
考えてみました。

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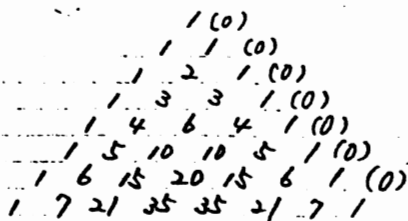
要素の積の形による	
数列の分析の例	
台形数による作らぬ	-
数列について	-

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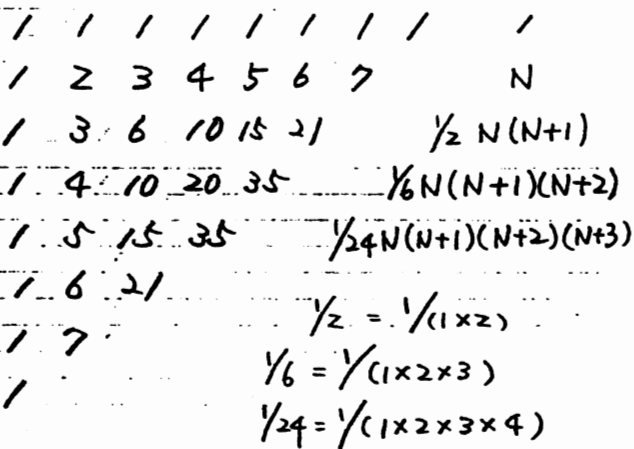
2008. 1. 29

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パスカルの三角形について



数字をななめにひらくと

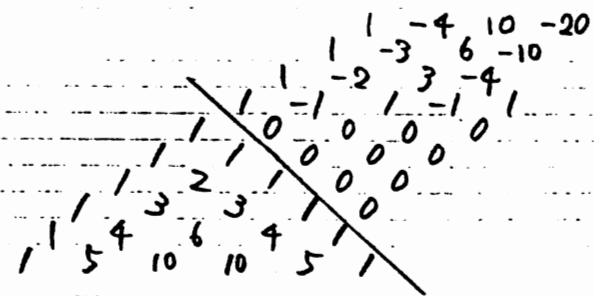


$$\begin{aligned}\frac{1}{2} &= \frac{1}{1 \times 2} \\ \frac{1}{6} &= \frac{1}{1 \times 2 \times 3} \\ \frac{1}{24} &= \frac{1}{1 \times 2 \times 3 \times 4}\end{aligned}$$

階差の項数列を使つて

$$\begin{aligned}0. 1. 0 &\rightarrow 1 \times (1) \rightarrow N \\ 0. 1. 1. 0 &\rightarrow \frac{1}{2} \times (1, 2) \rightarrow \frac{1}{2} N^2 + \frac{1}{2} N \\ 0. 1. 2. 1. 0 &\rightarrow \frac{0}{6} \frac{2}{1} \frac{1}{1} \rightarrow \frac{1}{6} (1, 6, 6) \\ &\quad - \frac{1}{2} \frac{1}{1} \rightarrow \frac{1}{2} (1, 2) \\ &\quad \frac{1}{6} \frac{1}{1} \rightarrow \frac{1}{6} (1, 6, 6) \\ &\quad - \frac{1}{2} \frac{1}{1} \rightarrow \frac{1}{2} (1, 2) \\ &\quad \frac{1}{6} (1, 6, 6) + \frac{1}{2} (1, 2) = \frac{1}{6} N (N^2 + 3N + 2) \\ &\quad = \frac{1}{6} N (N+1)(N+2) \\ 0. 1. 3. 3. 1 &\rightarrow \frac{1}{24} \frac{1}{4} \frac{3}{12} \frac{1}{24} \frac{1}{4} \\ &\quad - \frac{1}{24} \frac{1}{4} \frac{3}{12} \frac{1}{24} \frac{1}{4} \rightarrow \frac{1}{24} (1, 14, 36, 24) \\ &\quad \frac{1}{6} \frac{1}{2} \frac{3}{6} \frac{1}{2} \\ &\quad - \frac{1}{24} \frac{1}{4} \frac{3}{12} \frac{1}{24} \frac{1}{4} \rightarrow \frac{1}{24} (1, 6, 6) \\ &\quad \frac{1}{24} \frac{1}{12} \frac{1}{12} \frac{1}{24}\end{aligned}$$

別の方法 (右上に数列をのける)



-4. -3. -2. -1. 0. 1. 2. 3. 4.  
0次 1次 2次 3次 Nを含む  
10. 6. 3. 1. 0. 0. 1. 3. 6. 10  
0次 2次 3次 2次式 N=0の時=0  
N=-1の時=5  
N.(N+1)を含む  
N=1の時 1x2=2は合計全体をZで割る  
 $\frac{1}{2} \cdot N \cdot (N+1)$

-20 -10 -4 -1 0 0 0 1 4  
0次 3次 3次式 N.(N+1).(N+2)  
1x2x3=6を含む  
 $\frac{1}{6} \cdot N \cdot (N+1) \cdot (N+2)$

$$\begin{aligned}\frac{1}{24} \frac{1}{12} \\ - \frac{1}{24} \frac{2}{24} \quad \frac{1}{24} (1, 2) \\ \frac{1}{24} (\frac{1}{4})\end{aligned}$$

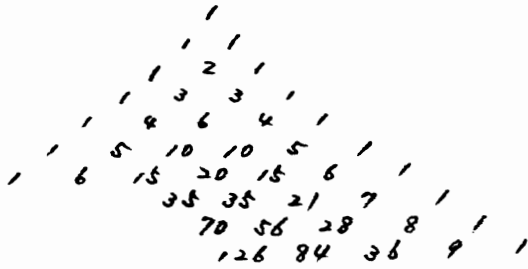
$$\frac{1}{24} N^4 + \frac{1}{4} N^3 + \frac{11}{24} N^2 + \frac{1}{4} N$$

$$\begin{aligned} \times 24 \\ \begin{array}{cccccc} 1 & 6 & 11 & 6 & & \\ 1 & 5 & 6 & & & \\ 1 & 4 & 6 & & & \\ 1 & 3 & 6 & & & \end{array} \times (1, 1) \\ \begin{array}{cccccc} 1 & 5 & 6 & & & \\ 1 & 4 & 6 & & & \end{array} \times (1, 2) \\ 1 \quad 3 \quad 6 \quad & \times (1, 3)\end{aligned}$$

$$\frac{1}{24} N \cdot (N+1) \cdot (N+2) \cdot (N+3)$$

三角数について

S(1)



S(1)-2

1 2 3 4 5 6 7

S(1)-3

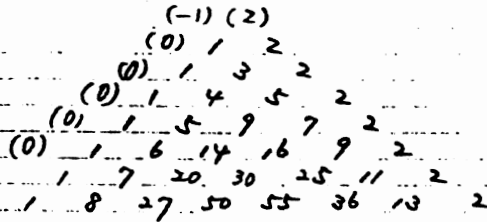
1 3 6 10 15 21 28

S(1)-4

1 4 10 20 35 56 84

S(1)-2       $\frac{1}{1}$     NS(1)-3       $\frac{1}{2}$     N(N+1)S(1)-4       $\frac{1}{3!}$     N(N+1)(N+2)

台形数 (頂点が 1, 2 の場合)



1, 4, 9, 16, 25, 36  $N^2$   
 1, 5, 14, 30, 55  $N^2$  の和  
 の数列 E 作) = E が 293

$2N - 1$

$N^2$

$\frac{1}{6} N(N+1)(2N+1)$

と の よ) の 規 則 性 が 示 された

0, 1, 4, 5, 2

$-\frac{1}{12} \frac{14}{12} \frac{3}{12} \frac{2}{12} \frac{1}{12} (1, 14, 36, 24)$

$\frac{1}{12} \frac{2}{6} 2$

$-\frac{1}{3} \frac{6}{3} \frac{4}{3} \frac{1}{3} (1, 6, 6)$

$\frac{7}{12} \frac{5}{6}$

$-\frac{5}{12} \frac{11}{12} \frac{5}{12} (1, 2)$

$\frac{2}{12} \frac{1}{6}$

$\frac{1}{6} (1)$

$$\begin{aligned}
 & \frac{1}{12} N^4 + \frac{1}{3} N^3 + \frac{5}{12} N^2 + \frac{1}{6} N \\
 &= \frac{1}{12} N(N^3 + 4N^2 + 5N + 2) \\
 &= \frac{1}{12} N(N+1)(N^2 + 3N + 2) \\
 &= \frac{1}{12} N(N+1)(N+1)(N+2)
 \end{aligned}$$

0, 1, 5, 9, 7, 2

$\frac{1}{60} \frac{3}{60} \frac{15}{60} \frac{24}{60} \frac{12}{60} \frac{1}{60} (1, 30, 150, 240, 120)$

$-\frac{1}{60} \frac{1}{2} \frac{5}{2} 4 2$

$\frac{5}{60} 4\frac{1}{2} 6\frac{1}{2} 3$

$\frac{1}{8} \frac{14}{8} \frac{3}{8} \frac{2}{8} \frac{1}{8} (1, 14, 36, 24)$

$-\frac{15}{120} 1\frac{3}{4} 4\frac{1}{2} 3$

$\frac{103}{120} 2\frac{3}{4} 2$

$\frac{1}{3} \frac{6}{3} \frac{4}{3} \frac{1}{3} (1, 6, 6)$

$-\frac{40}{120} 2 2$

$\frac{63}{120} \frac{3}{4}$

$\frac{3}{8} \frac{6}{8} \frac{3}{8} (1, 2)$

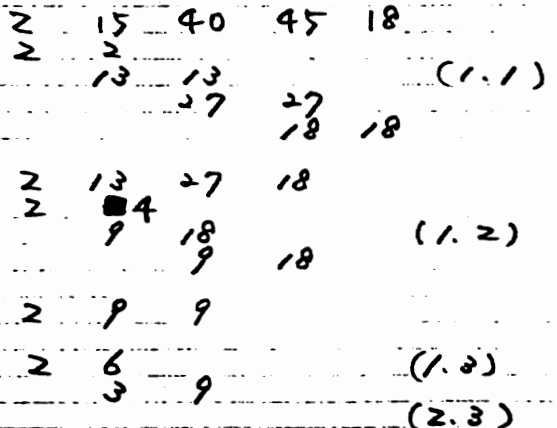
$-\frac{45}{120} \frac{3}{4}$

$\frac{18}{120}$

$\frac{3}{20} \frac{3}{20} (1)$

$\frac{1}{60} N^5 + \frac{1}{8} N^4 + \frac{1}{3} N^3 + \frac{3}{8} N^2 + \frac{3}{20} N$

X 120



$\frac{1}{120} N(N+1)(N+2)(N+3)(2N+3)$

$$2N - 1$$

$$N^2$$

$$\frac{1}{6} N(N+1)(2N+1)$$

$$\frac{1}{12} N(N+1)(N+1)(N+2)$$

$$\frac{1}{120} N(N+1)(N+2)(N+3)(2N+3)$$

整理すると

$$\frac{1}{1} (2N-1)$$

$$\frac{1}{(1 \times 2)} N(2N+0)$$

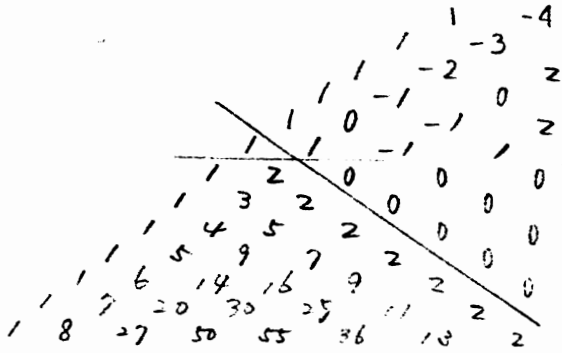
$$\frac{1}{(1 \times 2 \times 3)} N \cdot (N+1)(2N+1)$$

$$\frac{1}{(1 \times 2 \times 3 \times 4)} N \cdot (N+1)(N+2)(2N+2)$$

$$\frac{1}{(1 \times 2 \times 3 \times 4 \times 5)} N \cdot (N+1)(N+2)(N+3)(2N+3)$$

$S(1) - n$  の規則性を利用します。

頂上を 1 2 とする台形数列

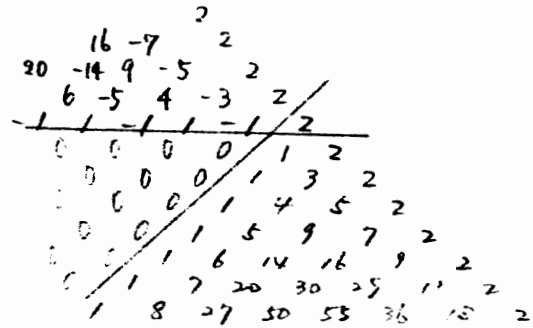


$$\frac{1}{1} (n+1) \quad (2.1)_1'$$

$$\frac{1}{1 \times 2} n(n+3) \quad (2.1)_1^3$$

$$\frac{1}{1 \times 2 \times 3} n(n+1)(n+5) \quad (2.1)_1^5$$

$$\frac{1}{1 \times 2 \times 3 \times 4} n(n+1)(n+2)(n+7) \quad (2.1)_1^7$$



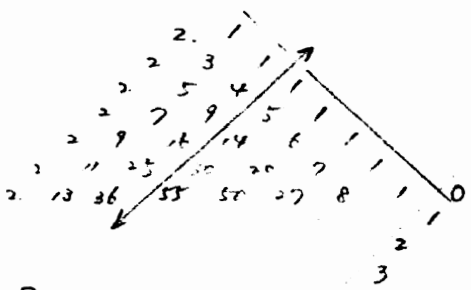
$$\frac{1}{1} (2n-1) \quad (2.1)_2^{-1}$$

$$\frac{1}{1 \times 2} n(2n+0) \quad (2.1)_2^0$$

$$\frac{1}{1 \times 2 \times 3} n(n+1)(2n+1) \quad (2.1)_2^1$$

$$\frac{1}{1 \times 2 \times 3 \times 4} n(n+1)(n+2)(2n+2) \quad (2.1)_2^2$$

台形数列 (2.1)



$$(2.1)_2^{-1} 2n-1 \quad (2n-1)$$

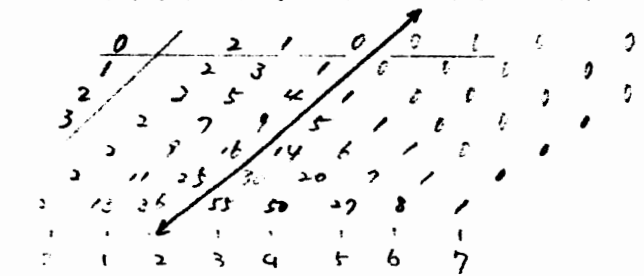
$$(2.1)_2^0 n^2 = \frac{1}{2} n(2n+0)$$

$$(2.1)_2^1 \frac{1}{3} n^3 + \frac{1}{2} n^2 + \frac{1}{6} n = \frac{1}{6} n(n+1)(2n+1)$$

$$(2.1)_2^2 \frac{1}{12} n^4 + \frac{1}{2} n^3 + \frac{1}{2} n^2 + \frac{1}{6} n = \frac{1}{24} n(n+1)(n+2)(2n+2)$$

$(2.1)_2^0 \rightarrow (2n+Q) \text{ a } Q \text{ の値}$   
 $\uparrow \rightarrow 2 \text{ の階乗}$   
 台形数列の 0 段

台形数列 別の見方



$$0 \quad 2$$

$$1 \quad 2n+1 \quad (2n+1)$$

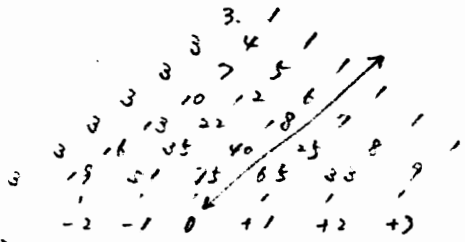
$$2 \quad n^2 = \frac{1}{2} (2n+0)$$

$$3 \quad \frac{1}{6} n(n-1)(2n-1)$$

$$4 \quad \frac{1}{24} n(n-1)(n-2)(2n-2)$$

$(2.1)_2^1 \rightarrow 2 \text{ の階乗}$

台形数列 (3.1)



$$(3.1)_3^{-2} \frac{1}{1} (3n-2)$$

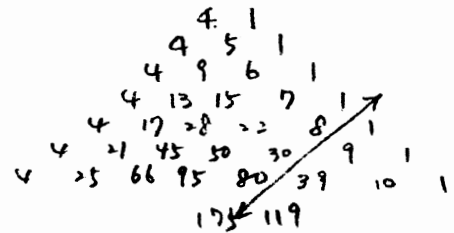
$$(3.1)_3^{-1} \frac{1}{1 \times 2} n(3n-1)$$

$$(3.1)_3^0 \frac{1}{1 \times 2 \times 3} n(n+1)(3n+0)$$

$$(3.1)_3^1 \frac{1}{1 \times 2 \times 3 \times 4} n(n+1)(n+2)(3n+1)$$

$$(3.1)_3^2 \frac{1}{1 \times 2 \times 3 \times 4 \times 5} n(n+1)(n+2)(n+3)(3n+2)$$

(4.1)



$$(4.1)_4^{-3} \frac{1}{1} (4n-3)$$

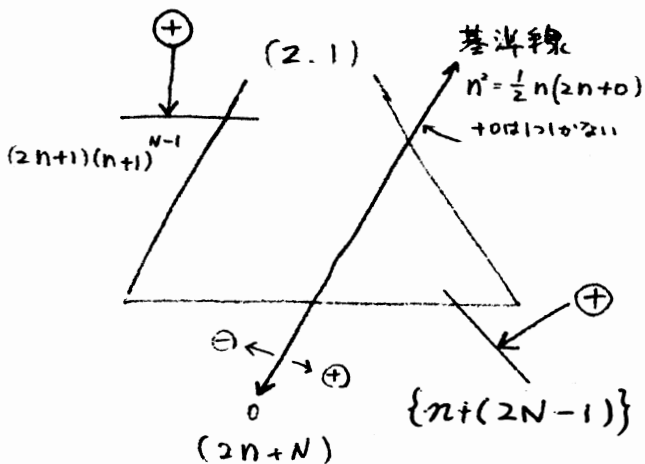
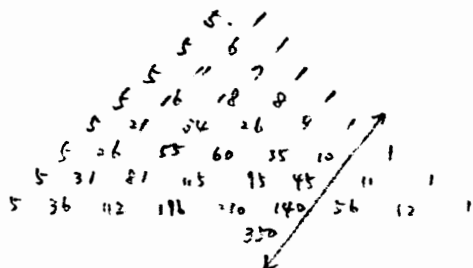
$$(4.1)_4^{-2} \frac{1}{1 \times 2} n(4n-2)$$

$$(4.1)_4^{-1} \frac{1}{1 \times 2 \times 3} n(n+1)(4n-1)$$

$$(4.1)_4^0 \frac{1}{1 \times 2 \times 3 \times 4} n(n+1)(n+2)(4n+0)$$

$$(4.1)_4^1 \frac{1}{1 \times 2 \times 3 \times 4 \times 5} n(n+1)(n+2)(n+3)(4n+1)$$

台形数列 (5.1)



基准线 (+0) 上 0 数值

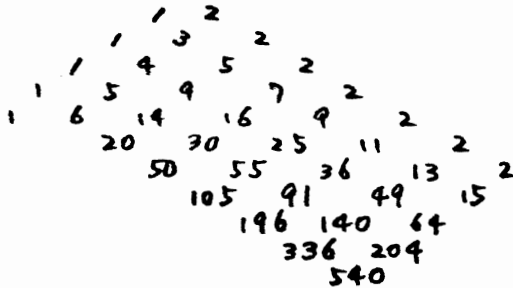
1.1	1	2	3	4	5	6
2.1	1	4	9	16	25	36
3.1	1	6	18	40	75	126
4.1	1	8	30	80	175	
5.1	1	10	45	140	350	

1x	2x	3x	4x	5x	6x
(1, 1)	(1, 3)	(1, 6)	(1, 10)	(1, 15)	(1, 21)
(1, 1)	(1, 3)	(1, 6)	(1, 10)	(1, 15)	(1, 21)
(1, 1)	(1, 3)	(1, 6)	(1, 10)	(1, 15)	(1, 21)
(1, 1)	(1, 3)	(1, 6)	(1, 10)	(1, 15)	(1, 21)
(1, 1)	(1, 3)	(1, 6)	(1, 10)	(1, 15)	(1, 21)

$$(1.1) \oplus \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}$$

台形数に2112

D(1.2)



D(1.2) - 1

1 . 3 . 5 . 7 . 9 . 11 . 13

D(1.2) - 2

1 . 4 . 9 . 16 . 25 . 36 . 49

D(1.2) - 3

1 . 5 . 14 . 30 . 55 . 91 . 140

D(1.2) - 4

1 . 6 . 20 . 50 . 105 . 196 . 336

D(1.2) - 1

$\frac{1}{1} (2N - 1)$

D(1.2) - 2

$\frac{1}{2} N(2N + 0)$

D(1.2) - 3

$\frac{1}{3} N(N+1)(2N+1)$

D(1.2) - 4

$\frac{1}{4} N(N+1)(N+2)(2N+2)$

続きを求めます。

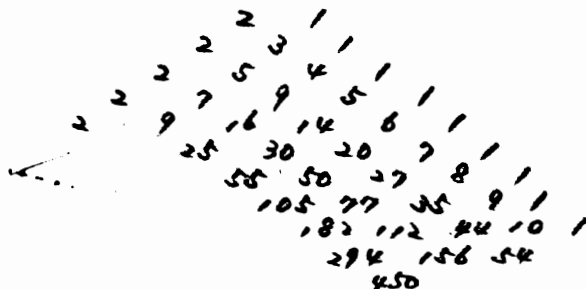
D(1.2) - 5

$\frac{1}{5} N(N+1)(N+2)(N+3)(2N+3)$

D(1.2) - 6

$\frac{1}{6} N(N+1)(N+2)(N+3)(N+4)(2N+4)$

D(2.1)



D(2.1) - 1

2 . 3 . 4 . 5 . 6 . 7 . 8

D(2.1) - 2

2 . 5 . 9 . 14 . 20 . 27 . 35

D(2.1) - 3

2 . 7 . 16 . 30 . 50 . 77 . 112

D(2.1) - 4

2 . 9 . 25 . 55 . 105 . 182 . 294

D(2.1) - 1

$\frac{1}{1} (N+1)$

D(2.1) - 2

$\frac{1}{2} N(N+3)$

D(2.1) - 3

$\frac{1}{3} N(N+1)(N+5)$

D(2.1) - 4

$\frac{1}{4} N(N+1)(N+2)(N+7)$

続きを求めます。

D(2.1) - 5

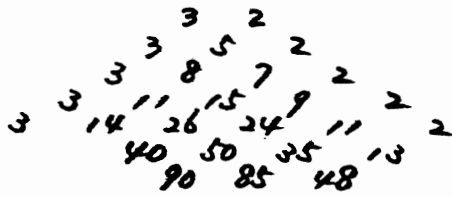
$\frac{1}{5} N(N+1)(N+2)(N+3)(N+9)$

D(2.1) - 6

$\frac{1}{6} N(N+1)(N+2)(N+3)(N+4)(N+11)$



D(3,2)



D(3,2)-1

$$3 \quad 5 \quad 7 \quad 9 \quad 11 \quad 13$$

$$\frac{1}{1} (2N+1)$$

D(3,2)-2

$$3 \quad 8 \quad 15 \quad 24 \quad 35 \quad 48$$

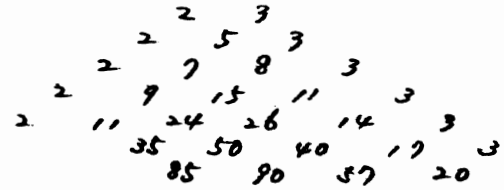
$$\frac{1}{2} N(2N+4)$$

D(3,2)-3

$$3 \quad 11 \quad 26 \quad 50 \quad 85$$

$$\frac{1}{3!} N(N+1)(2N+7)$$

D(2,3)



D(2,3)-1

$$2 \quad 5 \quad 8 \quad 11 \quad 14 \quad 17$$

$$\frac{1}{1} (3N-1)$$

D(2,3)-2

$$2 \quad 7 \quad 15 \quad 26 \quad 40 \quad 57$$

$$\frac{1}{2} N(3N+1)$$

D(2,3)-3

$$2 \quad 9 \quad 24 \quad 50 \quad 90$$

$$\frac{1}{3!} N(N+1)(3N+3)$$

D(A,B)-n の構造

-n

$$(1) \frac{1}{1} (BN+x)$$

$$x = A - B$$

$$(2) \frac{1}{2} N(BN+x+A)$$

$$(3) \frac{1}{3!} N(N+1)(BN+x+2A)$$

$$(4) \frac{1}{4!} N(N+1)(N+2)$$

$$(BN+x+3A)$$

台形数 (1.6.6 の場合)

- $\frac{1}{2} (6N^2 - 6N + 2)$
- $\frac{1}{6} N(6N^2 + 0 + 0)$
- $\frac{1}{24} N(N+1)(6N^2 + 6N + 0)$
- $\frac{1}{120} N(N+1)(N+2)(6N^2 + 12N + 2)$
- $\frac{1}{720} N(N+1)(N+2)(N+3)(6N^2 + 18N + 6)$
- $\frac{1}{5040} N(N+1)(N+2)(N+3)(N+4)$   
( $6N^2 + 24N + 12$ )
- $\frac{1}{40320} N(N+1)(N+2)(N+3)(N+4)(N+5)$   
( $6N^2 + 30N + 20$ )

S(1)- $\pi$  D(1.2)- $\pi$   
の規則性を利用して求めます。

D(1.6.6)- $\pi$

n 場合の  $\pi$

- 2  $\rightarrow (-2) \times (-1)$
- 0  $\rightarrow (-1) \times 0$
- 0  $\rightarrow 0 \times 1$
- 2  $\rightarrow 1 \times 2$
- 6  $\rightarrow 2 \times 3$
- 12  $\rightarrow 3 \times 4$
- 20  $\rightarrow 4 \times 5$

の規則性が表われる。

D(1.6.6)

1 6 6  
1 7 12 6  
1 8 19 18 6  
1 9 27 37 24 6  
1 10 36 64 61 30 6

D(1.6.6)-1

1. 7. 19. 37. 61. 91

$\frac{1}{2} (6N^2 - 6N + 2)$

D(1.6.6)-2

1. 8. 27. 64. 125. 216

$\frac{1}{3!} N(6N^2 + 0 + 0)$

D(1.6.6)-3

1. 9. 36. 100. 225

$\frac{1}{4!} N(N+1)(6N^2 + 6N + 0)$

D(1.6.6)-4

$\frac{1}{5!} N(N+1)(N+2)(6N^2 + 12N + 2)$

D(A,B,B)- $\pi$  の構造

- $\pi$

- (1)  $\frac{1}{2} (BN^2 - BN + \alpha)$
- (2)  $\frac{1}{3!} N(BN^2 + 0 + \alpha)$
- (3)  $\frac{1}{4!} N(N+1)(BN^2 + BN + \alpha)$
- (4)  $\frac{1}{5!} N(N+1)(N+2)$   
( $BN^2 + 2BN + \alpha$ )

$\alpha$  は A を使って決定して下さい。

D(1.1.1)



D(1.1.1) - 1

1. 2. 4. 7. 11. 16

$$\frac{1}{2} (N^2 - N + 2)$$

D(1.1.1) - 2

1. 3. 7. 14. 25

$$\frac{1}{3} N (N^2 + 0 + 5)$$

D(1.1.1) - 3

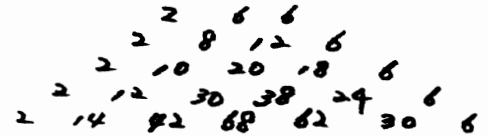
1. 4. 11. 25

$$\frac{1}{4} N (N+1) (N^2 + N + 10)$$

D(1.1.1) - 4

$$\frac{1}{5} N (N+1) (N+2) (N^2 + 2N + 17)$$

D(2.6.6)



D(2.6.6) - 1

2. 8. 20. 38. 62. 92. 128

$$\frac{1}{2} (6N^2 - 6N + 4)$$

D(2.6.6) - 2

2. 10. 30. 68. 130. 222

$$\frac{1}{3} N (6N^2 + 0 + 6)$$

D(2.6.6) - 3

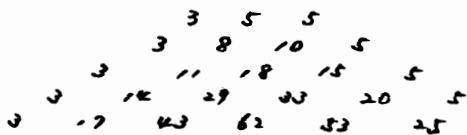
2. 12. 42. 110. 240

$$\frac{1}{4} N (N+1) (6N^2 + 6N + 12)$$

D(2.6.6) - 4

$$\frac{1}{5} N (N+1) (N+2) (6N^2 + 12N + 22)$$

D(3.5.5)



D(3.5.5) - 1

3. 8. 18. 33. 53. 78

$$\frac{1}{2} (5N^2 - 5N + 6)$$

D(3.5.5) - 2

3. 11. 29. 62. 115. 193

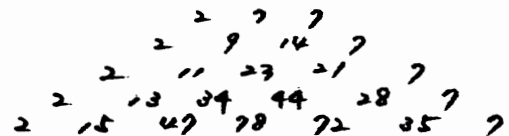
$$\frac{1}{3} N (5N^2 + 0 + 13)$$

D(3.5.5) - 3

3. 14. 43. 105. 220. 413

$$\frac{1}{4} N (N+1) (5N^2 + 5N + 26)$$

D(2.7.7)



D(2.7.7) - 1

2. 9. 23. 44. 72. 107

$$\frac{1}{2} (7N^2 - 7N + 4)$$

D(2.7.7) - 2

2. 11. 34. 78. 150

$$\frac{1}{3} N (7N^2 + 0 + 5)$$

D(2.7.7) - 3

2. 13. 47. 125

$$\frac{1}{4} N (N+1) (7N^2 + 7N + 10)$$

D(3.5.5)

D(3.5.5) - 1

3. 8. 18. 33. 53. 78

$$\frac{1}{2} (5N^2 - 5N + 6)$$

D(3.5.5) - 2

3. 11. 29. 62. 115. 193

$$\frac{1}{3} N (5N^2 + 0 + 13)$$

D(3.5.5) - 3

3. 14. 43. 105. 220. 413

$$\frac{1}{4} N (N+1) (5N^2 + 5N + 26)$$

D(2.7.7)

D(2.7.7) - 1

2. 9. 23. 44. 72. 107

$$\frac{1}{2} (7N^2 - 7N + 4)$$

D(2.7.7) - 2

2. 11. 34. 78. 150

$$\frac{1}{3} N (7N^2 + 0 + 5)$$

D(2.7.7) - 3

2. 13. 47. 125

$$\frac{1}{4} N (N+1) (7N^2 + 7N + 10)$$