

$M=1$	$\frac{1}{2} N^2 + \frac{1}{2} N$				
$M=2$	$\frac{1}{3} N^3 + \frac{1}{2} N^2 + \frac{1}{6} N$				
$M=3$	$\frac{1}{4} N^4 + \frac{1}{2} N^3 + \frac{1}{4} N^2$				
$M=4$	$\frac{1}{5} N^5 + \frac{1}{2} N^4 + \frac{1}{3} N^3$	$-\frac{1}{30} N$			
$M=5$	$\frac{1}{6} N^6 + \frac{1}{2} N^5 + \frac{5}{12} N^4$	$-\frac{1}{12} N^2$			
$M=6$	$\frac{1}{7} N^7 + \frac{1}{2} N^6 + \frac{1}{2} N^5$	$-\frac{1}{6} N^3$	$+\frac{1}{42} N$		
$M=7$	$\frac{1}{8} N^8 + \frac{1}{2} N^7 + \frac{7}{12} N^6$	$-\frac{7}{24} N^4$	$+\frac{1}{12} N^2$		
$M=8$	$\frac{1}{9} N^9 + \frac{1}{2} N^8 + \frac{2}{3} N^7$	$-\frac{7}{15} N^5$	$+\frac{2}{9} N^3$	$-\frac{1}{30} N$	
$M=9$	$\frac{1}{10} N^{10} + \frac{1}{2} N^9 + \frac{3}{4} N^8$	$-\frac{7}{10} N^6$	$+\frac{1}{2} N^4$	$-\frac{3}{20} N^2$	
$M=10$	$\frac{1}{11} N^{11} + \frac{1}{2} N^{10} + \frac{5}{6} N^9$	$-N^7$	$+N^5$	$-\frac{1}{2} N^3$	$+\frac{5}{66} N$

1からNまでのM乗数の和  
(数値はポケット・コンピュータを使用)

M = 1  
 N = 10 55  
 N = 100 5050  
 N = 1000 500500

M = 2  
 N = 10 385  
 N = 100 338350  
 N = 1000 333833500

M = 3  
 N = 10 3025  
 N = 100 25502500

M = 4  
 N = 10 25333  
 N = 100 2050333330

M = 5  
 N = 10 220825  
 N = 100  $1.717083325 \times 10^{11}$   
 N = 1000  $1.671670833 \times 10^{17}$

M = 6  
 N = 10  
 N = 100  $1.479071412 \times 10^{13}$   
 N = 1000  $1.433576429 \times 10^{20}$

M = 7  
 N = 10 18080425  
 N = 100  $1.300583304 \times 10^5$   
 N = 1000  $1.255005833 \times 10^{23}$

M = 8  
 N = 10 167731333  
 N = 100  $1.161777731 \times 10^{17}$   
 N = 1000  $1.116117778 \times 10^{26}$

M = 9  
 N = 10 1574304985  
 N = 100  $1.05074993 \times 10^{19}$   
 N = 1000  $1.0050075 \times 10^{29}$

M = 10  
 N = 10 14914341925  
 N = 100  $9.599241424 \times 10^{20}$   
 N = 1000  $9.140992424 \times 10^{31}$

## 第2章

素数を利用する乗算の打ち方

素因数分解の BASIC プログラム

と

M=4 の場合の計算例

## 素因数分解の BASIC プログラム

(CASIO FX-890P)

```

5 CLEAR
10 PRINT "インスタンカイ N?"
20 INPUT "N=" ; N
30 PRINT "N=" ; N
40 A = INT (SQR N)
50 FOR I = 2 TO A
60 B = N / I
70 IF FRAC (B) = 0 THEN 100
80 NEXT I
90 PRINT N ; " (END)" : GOTO 5
100 PRINT I ; "*" ;
110 N = B
120 GOTO 40

```

3

4

M=4 の場合

(N)

1	1	1	
2	16	17	
3	81	82	$2 \cdot 7^2$
4	256	257	$2 \cdot 3 \cdot 59$
5	625	626	$7 \cdot 89$
6	1296	1297	$5^2 \cdot 7 \cdot 13$
7	2401	2402	$2^2 \cdot 7 \cdot 167$
8	4096	4097	$2^2 \cdot 3 \cdot 17 \cdot 43$
9	6561	6562	$3 \cdot 19 \cdot 289$
10	10000	10001	$7^2 \cdot 11 \cdot 47$

(N)

1		
2	$2 \cdot 7^2$	$7 = 2N + 1$
3	$2 \cdot 3 \cdot 59$	
4	$11 \cdot 89$	$11 = 2N + 1$
5	$5^2 \cdot 7 \cdot 13$	$7 = N + 1$ $13 = 2N + 1$
6	$2^2 \cdot 7 \cdot 167$	$7 = N$
7	$2^2 \cdot 3 \cdot 17 \cdot 43$	$17 = 2N + 1$
8	$3 \cdot 19 \cdot 289$	$19 = 2N + 1$
9	$7^2 \cdot 11 \cdot 47$	$11 = N + 1$

$$N \cdot (N+1) \cdot (2N+1)$$

の要素があることがわかる。

(N)		$N \cdot (N+1) \cdot (2N+1)$
2	17	$2 \cdot 3 \cdot 5 = 30$
3	$7 \times \textcircled{5}$ 2.7	$3 \cdot 4 \cdot 7 = 84$ $30 \div 6 = \textcircled{5}$
4	59 2.3	$4 \cdot 5 \cdot 9 = 180$ $2 \cdot 5 \cdot 3 = 30$
5	89 "	$5 \cdot 6 \cdot 11 = 330$ $5 \cdot 6 = 30$
6	$5 \times 5 \times \textcircled{5}$ 7.13	$6 \cdot 7 \cdot 13 = 546$ $6 = 6$ $30 \div 6 = \textcircled{5}$
7	167 2.7	$7 \cdot 8 \cdot 15 = 840$ $2 \cdot 15 = 30$
8	$43 \times \textcircled{5}$ 2.3.17	$8 \cdot 9 \cdot 17 = 1224$ $2 \cdot 3 = 6$ $30 \div 6 = \textcircled{5}$
9	269 3.19	$9 \cdot 10 \cdot 19 = 1710$ $3 \cdot 10 = 30$
10	$7 \times 47$ 7.11	$10 \cdot 11 \cdot 21 = 2310$ $10 \cdot 3 = 30$

2	17	17
3	$7 \times 5$	35
4	59	59
5	89	89
6	$5 \times 5 \times 5$	125
7	167	167
8	$43 \times 5$	215
9	269	269
10	$7 \times 47$	329

$N=10 \quad 329 \rightarrow (3N^2 + 3N - 1)$   
 $N=2 \quad 3 \cdot 4 + 3 \cdot 2 - 1 = 12 + 6 - 1 = 17$   
 $\frac{1}{30} \cdot N \cdot (N+1) \cdot (2N+1) \cdot (3N^2 + 3N - 1)$

$N \cdot (N+1) \cdot (2N+1)$   
 $1 \times 2 \times 3 = 6$   
 $N=2 \quad 17 \times 1 \text{ などは}$   
 $2 \times 3 \times 5 = 30$   
 $30 \div 6 = 5$   
 $N=10 \quad 329 \text{ を使って}$   
 $3N^2 + 3N - 1$   
 $N=1 \quad 3 + 3 - 1 = 5$

$(N) = 3 \text{ の場合}$   
 $98 = 2 \cdot 7 \cdot \textcircled{2}$   
 $N \cdot (N+1) \cdot (2N+1)$   
 $3 \cdot 4 \cdot 7$   
 $\textcircled{2} \times \textcircled{7}$   
 $3 \times 2 = 6$   
 $30 \div 6 = \textcircled{5}$   
 $7 \times \textcircled{5}$   
 $\boxed{35}$   
 $\left( \begin{array}{l} \underline{30} \text{ は } (N)=2 \text{ の場合} \\ \text{の } 2 \times 3 \times 5 = \underline{30} \\ \text{による得る数値} \end{array} \right)$

$$M=1 \quad \frac{1}{2} \cdot N \cdot (N+1)$$

$$M=2 \quad \frac{1}{6} \cdot N \cdot (N+1) (2N+1)$$

$$M=3 \quad \frac{1}{4} \cdot N^2 (N+1)^2$$

$$M=4 \quad \frac{1}{30} N \cdot (N+1) (2N+1) (3N^2+3N-1)$$

$$M=5 \quad \frac{1}{12} N^2 (N+1)^2 (2N^2+2N-1)$$

$$M=6 \quad \frac{1}{42} N \cdot (N+1) (2N+1) (3N^2+6N^3-3N+1)$$

規則性があるか?

$N=10$  の数値の素因数分解の表

$$M=1 \quad \underline{5} \times \underline{11}$$

$$M=2 \quad \underline{5} \times \underline{7} \times \underline{11}$$

$$M=3 \quad \underline{5} \times \underline{5} \times \underline{11} \times \underline{11}$$

$$M=4 \quad \underline{7} \times \underline{7} \times \underline{11} \times \underline{47}$$

$$M=5 \quad \underline{5} \times \underline{5} \times \underline{11} \times \underline{11} \times \underline{73}$$

$$M=6 \quad \underline{5} \times \underline{11} \times \underline{13} \times \underline{2767}$$

$$M=7 \quad \underline{5} \times \underline{5} \times \underline{11} \times \underline{11} \times \underline{5997}$$

$$M=8 \quad \underline{7} \times \underline{11} \times \underline{17} \times \underline{97} \times \underline{1321}$$

$$M=9 \quad \underline{5} \times \underline{11} \times \underline{11} \times \underline{109} \times \underline{23873}$$

$$5 \rightarrow \frac{1}{2} N$$

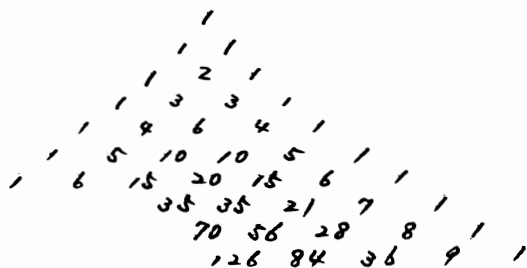
$$11 \rightarrow (N+1)$$

$$7 \rightarrow \frac{1}{3} (2N+1)$$

といえるか?

## パスカルの三角数の拡張

## パスカルの三角数について



$$S(1)-1$$

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$$

$$S(1)-2$$

$$1 \quad 3 \quad 6 \quad 10 \quad 15 \quad 21 \quad 28$$

$$S(1)-3$$

$$1 \quad 4 \quad 10 \quad 20 \quad 35 \quad 56 \quad 84$$

$$S(1)-1$$

$$\frac{1}{1} \quad N$$

$$S(1)-2$$

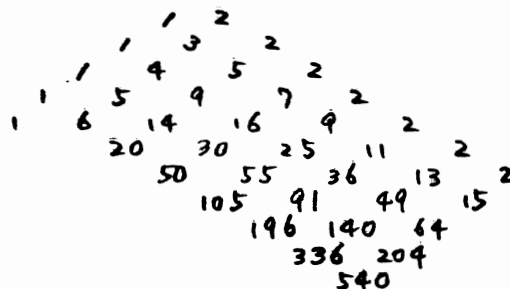
$$\frac{1}{2} \quad N(N+1)$$

$$S(1)-3$$

$$\frac{1}{3!} \quad N(N+1)(N+2)$$

## 台形数について

$$D(1,2)$$



$$D(1,2)-1$$

$$1 \quad 3 \quad 5 \quad 7 \quad 9 \quad 11 \quad 13$$

$$D(1,2)-2$$

$$1 \quad 4 \quad 9 \quad 16 \quad 25 \quad 36 \quad 49$$

$$D(1,2)-3$$

$$1 \quad 5 \quad 14 \quad 30 \quad 55 \quad 91 \quad 140$$

$$D(1,2)-4$$

$$1 \quad 6 \quad 20 \quad 50 \quad 105 \quad 196 \quad 336$$

$$D(1,2)-1$$

$$\frac{1}{1} \quad (2N-1)$$

$$D(1,2)-2$$

$$\frac{1}{2} \quad N(2N+0)$$

$$D(1,2)-3$$

$$\frac{1}{3!} \quad N(N+1)(2N+1)$$

$$D(1,2)-4$$

$$\frac{1}{4!} \quad N(N+1)(N+2)(2N+2)$$

続きを求めます。

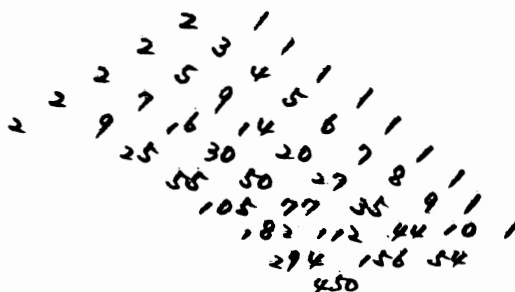
$$D(1,2)-5$$

$$\frac{1}{5!} \quad N(N+1)(N+2)(N+3)(2N+3)$$

$$D(1,2)-6$$

$$\frac{1}{6!} \quad N(N+1)(N+2)(N+3)(N+4)(2N+4)$$

$$D(2,1)$$



$$D(2,1)-1$$

$$2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

$$D(2,1)-2$$

$$2 \quad 5 \quad 9 \quad 14 \quad 20 \quad 27 \quad 35$$

$$D(2,1)-3$$

$$2 \quad 7 \quad 16 \quad 30 \quad 50 \quad 77 \quad 112$$

$$D(2,1)-4$$

$$2 \quad 9 \quad 25 \quad 55 \quad 105 \quad 182 \quad 294$$

$$D(2,1) - 1$$

$$\frac{1}{1} (N+1)$$

$$D(2,1) - 2$$

$$\frac{1}{2} N(N+3)$$

$$D(2,1) - 3$$

$$\frac{1}{3!} N(N+1)(N+5)$$

$$D(2,1) - 4$$

$$\frac{1}{4!} N(N+1)(N+2)(N+7)$$

続々求ます。

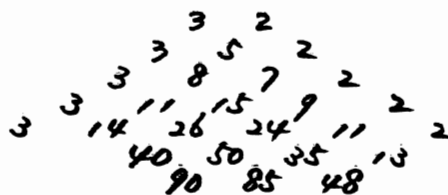
$$D(2,1) - 5$$

$$\frac{1}{5!} N(N+1)(N+2)(N+3)(N+9)$$

$$D(2,1) - 6$$

$$\frac{1}{6!} N(N+1)(N+2)(N+3)(N+4)(N+11)$$

$$D(3,2)$$



$$D(3,2) - 1$$

$$3 \ 5 \ 7 \ 9 \ 11 \ 13$$

$$\frac{1}{1} (2N+1)$$

$$D(3,2) - 2$$

$$3 \ 8 \ 15 \ 24 \ 35 \ 48$$

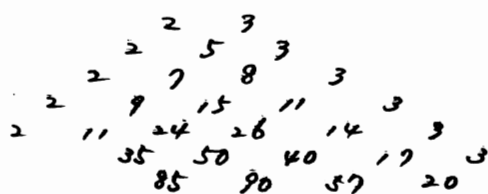
$$\frac{1}{2} N(2N+4)$$

$$D(3,2) - 3$$

$$3 \ 11 \ 26 \ 50 \ 85$$

$$\frac{1}{3!} N(N+1)(2N+7)$$

$$D(2,3)$$



$$D(2,3) - 1$$

$$2 \cdot 5 \cdot 8 \cdot 11 \cdot 14 \cdot 17$$

$$\frac{1}{1} (3N-1)$$

$$D(2,3) - 2$$

$$2 \cdot 7 \cdot 15 \cdot 26 \cdot 40 \cdot 57$$

$$\frac{1}{2} N(3N+1)$$

$$D(2,3) - 3$$

$$2 \cdot 9 \cdot 24 \cdot 50 \cdot 90$$

$$\frac{1}{3!} N(N+1)(3N+3)$$

$$D(1,1,1)$$



$$D(1,1,1) - 1$$

$$1 \cdot 2 \cdot 4 \cdot 7 \cdot 11 \cdot 16$$

$$\frac{1}{2} (N^2 - N + 2)$$

$$D(1,1,1) - 2$$

$$1 \cdot 3 \cdot 7 \cdot 14 \cdot 25$$

$$\frac{1}{3!} N(N^2 + 0 + 5)$$

$$D(1,1,1) - 3$$

$$1 \cdot 4 \cdot 11 \cdot 25$$

$$\frac{1}{4!} N(N+1)(N^2 + N + 10)$$

$$D(1,1,1) - 4$$

$$\frac{1}{5!} N(N+1)(N+2)(N^2 + 2N + 17)$$

9

10

D (1. 6. 6)

$$\begin{array}{cccccc} & & & 1 & 6 & 6 \\ & & & 1 & 7 & 12 & 6 \\ & & 1 & 8 & 19 & 28 & 6 \\ & 1 & 9 & 27 & 37 & 24 & 6 \\ 1 & 10 & 36 & 64 & 61 & 30 & 6 \end{array}$$

D (1.6.6) - 1

1. 7. 19. 37. 61. 91

$$\frac{1}{2} (6N^2 - 6N + 2)$$

D (1.6.6) - 2

1. 8. 27. 64. 125. 216

$$\frac{1}{3!} N(6N^2 + 0 + 0)$$

D (1.6.6) - 3

1. 9. 36. 100. 225

$$\frac{1}{4!} N(N+1)(6N^2 + 6N + 0)$$

D (1.6.6) - 4

$$\frac{1}{5!} N(N+1)(N+2)(6N^2 + 12N + 2)$$

D (2. 6. 6)

$$\begin{array}{cccccc} & & & & 2 & 6 & 6 \\ & & & & 2 & 8 & 12 & 6 \\ & & & 2 & 10 & 20 & 18 & 6 \\ & 2 & 12 & 30 & 38 & 24 & 6 \\ 2 & 14 & 42 & 68 & 62 & 30 & 6 \end{array}$$

D (2.6.6) - 1

2. 8. 20. 38. 62. 92. 128

$$\frac{1}{2} (6N^2 - 6N + 4)$$

D (2.6.6) - 2

2. 10. 30. 68. 130. 222

$$\frac{1}{3!} N(6N^2 + 0 + 6)$$

D (2.6.6) - 3

2. 12. 42. 110. 240

$$\frac{1}{4!} N(N+1)(6N^2 + 6N + 12)$$

D (2.6.6) - 4

$$\frac{1}{5!} N(N+1)(N+2)(6N^2 + 12N + 22)$$

11

12

D (3. 5. 5)

$$\begin{array}{cccccc} & & & & & 3 & 5 & 5 \\ & & & & & 3 & 8 & 10 & 5 \\ & & 3 & 11 & 18 & 15 & 5 \\ & 3 & 14 & 29 & 23 & 20 & 5 \\ 3 & 17 & 43 & 62 & 53 & 25 \end{array}$$

D (3.5.5) - 1

3. 8. 18. 33. 53. 78

$$\frac{1}{2} (5N^2 - 5N + 8)$$

D (3.5.5) - 2

3. 11. 29. 62. 115. 193

$$\frac{1}{3!} N(5N^2 + 0 + 13)$$

D (3.5.5) - 3

3. 14. 43. 105. 220. 413

$$\frac{1}{4!} N(N+1)(5N^2 + 5N + 26)$$

D (2. 7. 7)

$$\begin{array}{cccccc} & & & & & & 2 & 7 & 7 \\ & & & & & & 2 & 9 & 14 & 7 \\ & & & 2 & 11 & 23 & 21 & 7 \\ & 2 & 13 & 34 & 44 & 28 & 7 \\ 2 & 15 & 47 & 78 & 72 & 35 & 7 \end{array}$$

D (2.7.7) - 1

2. 9. 23. 44. 72. 107

$$\frac{1}{2} (7N^2 - 7N + 4)$$

D (2.7.7) - 2

2. 11. 34. 78. 150

$$\frac{1}{3!} N(7N^2 + 0 + 5)$$

D (2.7.7) - 3

2. 13. 47. 125

$$\frac{1}{4!} N(N+1)(7N^2 + 7N + 10)$$